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# Quantal Global Symmetry for a Gauge-Invariant System

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Based on the configuration-space generating functional obtained by using the Faddeev–Popov trick for a gauge-invariant system, the Ward identities for global transformation are derived. The conservation laws at the quantum level for global symmetry transformation are also deduced. A preliminary application of the present formulation to non-Abelian Chern–Simons (CS) theory is given. The Ward identity and quantal BRS charge under the BRS transformation are deduced. The quantal conserved angular momentum is obtained and the fractional spin for CS theories is discussed.

## 1. INTRODUCTION

The connection between global continuous symmetry and conservation laws is usually referred to as the first Noether (1918) theorem in the classical theories which are formulated in terms of Lagrange's variables in configuration space. The classical symmetry properties of the system in canonical formalism have been given in previous work (Li, 1994). Numerous recent works on (1 + 2)-dimensional gauge theories with Chern–Simons (CS) terms in the Lagrangian have revealed the occurrence of fractional spin and statistics (Banerjee, 1993; Kim and Park, 1994). They have attracted much attention due to their possible relevance to condensed matter phenomena, especially to the fractional quantum Hall effect and high-$T_c$ superconductivity (Lerda, 1992). In those papers the angular momentum which implies the fractional spin was deduced by using a classical Noether theorem. However, it is not clear whether those results are valid at the quantum level. Therefore one needs to discuss the quantal conservation laws for a system.

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Ward identities (or Ward–Takahashi identities) and their generalization play an important role in modern quantum field theories (Li, 1993). These identities have been generalized to the supersymmetry (Joglekar, 1991) and superstring theories (Danilov, 1991) and other problems. Ward identities are usually related to the local invariance of a system. Global symmetry, such as Lorentz invariance, conformal symmetry, BRS and BRST invariance, and supersymmetry, etc., has more fundamental importance in modern physics. Here the global symmetry of a gauge-invariant system at the quantum level will be further studied. The generalized Ward identities and quantal conservation laws under the global transformation will be established.

We must consider global symmetries when dealing with a quantum system. This is clear from the path-integral formulation, where the main ingredient is the classical action together with the measure in the space of field configurations. The phase-space path integrals are more basic than configuration-space path integrals; the latter provide a Hamiltonian quadratic in canonical momenta, whereas the former apply to arbitrary Hamiltonians (Mizrahi, 1978). Thus, the phase-space form of the path integral is a necessary precursor to the configuration form. In certain integrable cases, the phase-space path integral can be simplified by carrying out explicit integration over canonical momenta. Then, the phase-space path integral can be represented in the form of a path integral only over the coordinates (or field variables) of the expression containing a certain Lagrangian (or effective Lagrangian) in configuration space. In more general cases, especially for the constrained Hamiltonian system with complicated constraints, it is very difficult or even impossible to carry out the integration over the canonical momenta (Nishikawa, 1993). In these cases the Ward identities cannot be derived via a generating functional with a Lagrangian (or effective Lagrangian) in configuration space as in the traditional treatment. The quantal local and global symmetries in canonical formalism for a system was developed in previous work (Li, 1995, 1996a,b).

Local gauge invariance is a central concept in modern field theory. A system with a gauge-invariant Lagrangian is subject to some inherent phase-space constraint which is, in fact, a constrained Hamiltonian system (Li, 1993). The path-integral quantization of this system can be formulated with aid of the Dirac formalism for a singular Lagrangian (Dirac, 1964) and the method of functional integration (Faddeev, 1970; Sanjanovic, 1976). However, for a gauge-invariant system one can conveniently use the Faddeev–Popov trick (Faddeev and Popov, 1967) to formulate its path-integral quantization in configuration space. In certain cases (for example, Yang–Mills theory), according to the path-integral quantization of the constrained Hamiltonian system, one can carry out explicit integration over momenta in the phase-space path integral which can be converted to the same results by using the Faddeev–Popov trick. Although the Faddeev–Popov trick is not a rigorous method, it is a simpler and more useful method for the gauge theories.

Based on the configuration-space generating functional of the Green functions which is derived by using the Faddeev–Popov method, the Ward identities for a gauge-invariant system is deduced under the global transformation in configuration space. The local transformation corresponding to the global symmetry transformation is considered and the conservation laws are obtained at the quantum level if the effective action is symmetric and the Jacobian of the corresponding transformation is equal to unity. We give a preliminary application to non-Abelian CS theory; the Ward identity for the BRS transformation is obtained. The quantal BRS conserved charge and quantal conserved angular momentum for non-Abelian CS fields coupled to scalar fields are deduced. It has been shown that the quantal conserved angular momentum differs from the classical Noether one in that one needs to take into account the contribution of angular momenta of ghost fields in non-Abelian CS theories. The fractional spin in Abelian CS theories is discussed.

## 2. WARD IDENTITIES FOR GLOBAL TRANSFORMATION

Consider a system described by the field variables $\varphi^\alpha(x)$ ($\alpha = 1, 2, \ldots, n$), where $\alpha$ denotes an index for different fields or different components of a field. The Lagrangian of the field is $\mathcal{L}(\varphi^\alpha, \varphi^\alpha_{,\mu})$, where $\varphi^\alpha_{,\mu} = \partial_\mu\varphi^\alpha$, $\partial_\mu = \partial/\partial x^\mu$, and $x = (t, \mathbf{x})$. It is supposed that this Lagrangian is invariant under the gauge transformation. We choose the following gauge conditions:

$$F^a[\varphi^\alpha] = 0 \qquad (a = 1, 2, \ldots, m) \tag{1}$$

Using the Faddeev–Popov trick, through the transformation of the functional integral, we formulate the path-integral quantization for a gauge-invariant system, and obtain the configuration-space generating functional of the Green function for this system (Faddeev and Slavnov, 1980)

$$Z[J, \bar{\xi}, \xi] = \int \mathcal{D}\varphi^\alpha\, \mathcal{D}\bar{C}_\alpha\, \mathcal{D}C_\alpha \exp\left\{i \int d^4x\, (\mathcal{L}_{\text{eff}} + J_\alpha\varphi^\alpha + \bar{\xi}^\alpha C_\alpha + \bar{C}_\alpha\xi^\alpha)\right\} \tag{2}$$

where

$$\mathcal{L}_{\text{eff}} = \mathcal{L} + \mathcal{L}_{\text{fix}} + \mathcal{L}_{\text{gh}} \tag{3}$$

$$\mathcal{L}_{\text{fix}} = -\frac{1}{2\alpha_0}(F^a[\varphi^\alpha])^2 \tag{4}$$

$$\mathcal{L}_{\text{gh}} = \bar{C}_\alpha M_F^{\alpha\beta} C_\beta \tag{5}$$

and $\alpha_0$ is a gauge parameter, $\overline{C}_\alpha(x)$ and $C_\alpha(x)$ are the auxiliary anticommuting scalar fields, and $M_F^{\alpha\beta}$ is determined from the gauge conditions (1) and the gauge transformation.

For the sake of simplicity, let us denote $\varphi = (\varphi^\alpha, \overline{C}_\alpha, C_\alpha)$ and $J = (J_\alpha, \overline{\xi}^\alpha, \xi^\alpha)$, and consider a global transformation in configuration space whose infinitesimal transformation is given by

$$\begin{cases} x^\mu = x^\mu + \epsilon_\sigma \tau^{\mu\sigma}(x, \varphi, \varphi_{,\mu}) \\ \varphi'(x') = \varphi(x) + \epsilon_\sigma \xi^\sigma(x, \varphi, \varphi_{,\mu}) \end{cases} \tag{6}$$

where $\epsilon_\sigma$ ($\sigma = 1, 2, \ldots, r$) are infinitesimal arbitrary parameters, and $\tau^{\mu\sigma}$ and $\xi^\sigma$ are some functions of $x$, $\varphi(x)$, and $\varphi_{,\mu}(x)$. Under the transformation (6), the variation of the effective action is given by

$$\delta I_{\text{eff}} = \int d^4x\, \epsilon_\sigma \left\{ \frac{\delta I_{\text{eff}}}{\delta \varphi} (\xi^\sigma - \varphi_{,\mu}\tau^{\mu\sigma}) + \partial_\mu \left[ \frac{\partial \mathcal{L}}{\partial \varphi_{,\mu}} (\xi^\sigma - \varphi_{,\mu}\tau^{\mu\sigma}) + \mathcal{L}_{\text{eff}} \tau^{\mu\sigma} \right] \right\} \tag{7}$$

where

$$\frac{\delta I_{\text{eff}}}{\delta \varphi} = \frac{\partial \mathcal{L}_{\text{eff}}}{\partial \varphi} - \partial_\mu \left( \frac{\partial \mathcal{L}_{\text{eff}}}{\partial \varphi_{,\mu}} \right) \tag{8}$$

It is supposed that the Jacobian of the transformation (6) is equal to unity. The generating functional (2) is invariant under the transformation (6). Thus, we have

$$\begin{aligned} Z[J] &= \int \mathcal{D}\varphi \exp\left\{ i(I_{\text{eff}} + \Delta I_{\text{eff}}) + i \int d^4x\, [J\varphi + \epsilon_\sigma(\xi^\sigma - \varphi_{,\mu}\tau^{\mu\sigma}) \right. \\ &\quad \left. + \epsilon_\sigma \partial_\mu (J\varphi\tau^{\mu\sigma})] \right\} \\ &= \int \mathcal{D}\varphi \exp\left\{ i \int d^4x\, (\mathcal{L}_{\text{eff}} + J\varphi) \right\} \left\{ 1 + i\Delta I_{\text{eff}} + i\epsilon_\sigma \int d^4x\, [J(\xi^\sigma - \varphi_{,\mu}\tau^{\mu\sigma}) \right. \\ &\quad \left. + \epsilon_\sigma \partial_\mu (J\varphi\tau^{\mu\sigma})] \right\} \\ &= \left\{ 1 + i\Delta I_{\text{eff}} + i\epsilon_\sigma \int d^4x\, [J(\xi^\sigma - \varphi_{,\mu}\tau^{\mu\sigma}) + \partial_\mu (J\varphi\tau^{\mu\sigma})] \right\}_{\varphi \to \delta/i\delta J} Z[J] \end{aligned} \tag{9}$$

Consequently, if the Jacobian of the transformation (6) is equal to unity, then the generating functional (2) satisfies

$$\int d^4x \left\{ \frac{\delta I_{\text{eff}}}{\delta\varphi} (\xi^\sigma - \varphi_{,\mu}\tau^{\mu\sigma}) + \partial_\mu \left[ \frac{\partial \mathcal{L}_{\text{eff}}}{\partial \varphi_{,\mu}} (\xi^\sigma - \varphi_{,\mu}\tau^{\mu\sigma}) + \mathcal{L}_{\text{eff}}\tau^{\mu\sigma} \right] + J(\xi^\sigma - \varphi_{,\mu}\tau^{\mu\sigma}) + \partial_\mu(J\varphi\tau^{\mu\sigma}) \right\}_{\varphi \to \delta/i\delta J} Z[J] = 0 \tag{10}$$

If the effective action is invariant under the transformation (6), then from (10) one obtains

$$\int d^4x \,\{J(\xi^\sigma - \varphi_{,\mu}\tau^{\mu\sigma}) + \partial_\mu(J\varphi\tau^{\mu\sigma})\}_{\varphi \to \delta/i\delta J} Z[J] = 0 \tag{11}$$

For the internal symmetry transformation $\tau^{\mu\sigma} = 0$, in this case expression (11) can be written as

$$\int d^4x \,[J\xi^\sigma(x, \delta/i\delta J(x), (1/i)\partial_\mu(\delta/\delta J(x))]Z[J] = 0 \tag{12}$$

Functionally differentiating (10), (11), or (12) with respect to exterior sources $J(x)$ many times and setting exterior sources equal to zero, $J(x) = 0$, one can obtain relationships among the Green functions. Thus, the expressions (10)–(12) are called the Ward identities under the global transformation in configuration space. The global transformation for a gauge-invariant system implies that there are these Ward identities.

## 3. QUANTAL CONSERVATION LAWS

Let us suppose that the effective action is invariant under the transformation (6); thus, the classical conservation laws can be derived by using the classical Noether theorem. Now the realization of a connection between the symmetry and conservation laws at the quantum level will be studied. Consider the following local transformation connected with the transformation (6):

$$\begin{cases} x^{\mu'} = x^\mu + \epsilon_\sigma(x)\tau^{\mu\sigma}(x, \varphi, \varphi_{,\mu}) \\ \varphi'(x') = \varphi(x) + \epsilon_\sigma(x)\xi^\sigma(x, \varphi, \varphi_{,\mu}) \end{cases} \tag{13}$$

where $\epsilon_\sigma(x)$ ($\sigma = 1, 2, \ldots, r$) are infinitesimal arbitrary functions; their values and derivatives vanish on the boundary of the space-time domain. Under the transformation (13), the variation of the effective action is given by

$$\Delta I_{\text{eff}} = \int d^4x\, \epsilon_\sigma(x) \left\{ \frac{\delta I_{\text{eff}}}{\delta\varphi} (\xi^\sigma - \varphi_{,\mu}\tau^{\mu\sigma}) + \partial_\mu \left[ \frac{\partial \mathcal{L}_{\text{eff}}}{\partial \varphi_{,\mu}} (\xi^\sigma - \varphi_{,\nu}\tau^{\mu\sigma}) + \mathcal{L}_{\text{eff}}\tau^{\mu\sigma} \right] \right\}$$
$$+ \int d^4x \left[ \frac{\partial \mathcal{L}_{\text{eff}}}{\partial \varphi_{,\mu}} (\xi^\sigma - \varphi_{,\nu}\tau^{\nu\sigma}) + \mathcal{L}_{\text{eff}}\tau^{\mu\sigma} \right] \partial_\mu \epsilon_\sigma(x) \tag{14}$$

Because the effective action is invariant under the global transformation (6) for the case $\epsilon_\sigma(x) = \epsilon_\sigma$ (parameters), the first integral in expression (14) is equal to zero. According to the boundary conditions of $\epsilon_\sigma(x)$, the expression (14) can be written as

$$\Delta I_{\text{eff}} = -\int d^4x\, \epsilon_\sigma(x) \partial_\mu \left[ \frac{\delta \mathcal{L}_{\text{eff}}}{\partial \varphi_{,\mu}} (\xi^\sigma - \varphi_{,\nu}\tau^{\nu\sigma}) + \mathcal{L}_{\text{eff}}\tau^{\mu\sigma} \right] \tag{15}$$

It is supposed that the Jacobian of the transformation (13) is equal to unity. The invariance of the generating functional (2) under the transformation (13) implies that $\delta Z/\delta\epsilon_\sigma(x)|_{\epsilon_\sigma(x)=0} = 0$. Substituting (13) and (15) into (2) and functionally differentiating with respect to $\epsilon_\sigma(x)$, we obtain

$$\int \mathcal{D}\varphi \left\{ \partial_\mu \left[ \frac{\partial \mathcal{L}_{\text{eff}}}{\partial \varphi_{,\mu}} (\xi^\sigma - \varphi_{,\nu}\tau^{\nu\sigma}) + \mathcal{L}_{\text{eff}}\tau^{\mu\sigma} \right] - J(\xi^\sigma - \varphi_{,\nu}\tau^{\nu\sigma}) \right\}$$
$$\times \exp\left\{ i \int d^4x\, (\mathcal{L}_{\text{eff}} + J\varphi) \right\} = 0 \tag{16}$$

Functionally differentiating (16) with respect to $J(x)$ $n$ times, we obtain

$$\int \mathcal{D}\varphi \left( \left\{ \partial_\mu \left[ \frac{\partial \mathcal{L}_{\text{eff}}}{\partial \varphi_{,\mu}} (\xi^\sigma - \varphi_{,\nu}\tau^{\nu\sigma}) + \mathcal{L}_{\text{eff}}\tau^{\mu\sigma} \right] - J(\xi^\sigma - \varphi_{,\nu}\tau^{\nu\sigma}) \right\} )\varphi(x_1) \cdots \varphi(x_n)$$
$$+ (-i) \sum_j \varphi(x_1) \cdots \varphi(x_{j-1})\varphi(x_{j+1}) \cdots \varphi(x_n) N^\sigma \delta(x - x_j) \Big)$$
$$\times \exp\left\{ i \int d^4x\, (\mathcal{L}_{\text{eff}} + J\varphi) \right\} = 0 \tag{17}$$

where

$$N^\sigma = -(\xi^\sigma - \varphi_{,\nu}\tau^{\nu\sigma}) \tag{18}$$

Let us set $J = 0$ in expression (17); we get

$$\left\langle 0 | T^* \left\{ \partial_\mu \left[ \frac{\partial \mathcal{L}_{\text{eff}}}{\partial \varphi_{,\mu}} (\xi^\sigma - \varphi_{,\nu} \tau^{\nu\sigma}) + \mathcal{L}_{\text{eff}} \tau^{\mu\sigma} \right] \right\} \varphi(x_1) \cdots \varphi(x_n) | 0 \right\rangle$$
$$= (-i) \sum_j \langle 0 | T^*[\varphi(x_1) \cdots \varphi(x_{j-1}) \cdots \varphi(x_{j+1}) \cdots \varphi(x_n) N^\sigma] | 0 \rangle \delta(x - x_j) \quad (19)$$

where the symbol $T^*$ stands for the covariantized $T$ product (Young, 1987). Fixing $t$ and letting

$$t_1, t_2, \ldots, t_m \to +\infty, \qquad t_{m+1}, t_{m+2}, \ldots, t_n \to -\infty$$

and using the reduction formula (Young, 1987), from (19), one has

$$\left\langle \text{out}, m | \left\{ \partial_\mu \left[ \frac{\partial \mathcal{L}_{\text{eff}}}{\partial \varphi_{,\mu}} (\xi^\sigma - \varphi_{,\nu} \tau^{\nu\sigma}) + \mathcal{L}_{\text{eff}} \tau^{\mu\sigma} \right] \right\} | n - m, in \right\rangle = 0 \quad (20)$$

Since $m$ and $n$ are arbitrary, this leads to

$$\partial_\mu \left[ \frac{\partial \mathcal{L}_{\text{eff}}}{\partial \varphi_{,\mu}} (\xi^\sigma - \varphi_{,\nu} \tau^{\nu\sigma}) + \mathcal{L}_{\text{eff}} \tau^{\mu\sigma} \right] = 0 \quad (21)$$

We take the integral of expression (21) on 3-dimensional space; if we assume that the fields have a configuration which vanishes rapidly at spatial infinity, using the Gauss theorem, we have

$$Q^\sigma = \int d^3x \left[ \frac{\partial \mathcal{L}_{\text{eff}}}{\partial \varphi_{,0}} (\xi^\sigma - \varphi_{,\nu} \tau^{\nu\sigma}) + \mathcal{L}_{\text{eff}} \tau^{0\sigma} \right] \quad (22)$$

Thus, we obtain the following theorem:

If the effective action of a gauge-invariant system is invariant under the global transformation (6) in configuration space and the Jacobian of the corresponding transformation (13) is equal to unity, then there are some conserved quantities (operators) for such a system. These conserved quantities at the quantum level correspond to the classical conserved charges derived from the classical Noether theorem (Li, 1993). This result holds true for the anomaly-free theories.

The conserved quantities (22) at the quantum level in configuration space hold for the case when the Faddeev–Popov method is valid for those gauge-invariant systems. In the more general case, one can study the canonical symmetry of the system to obtain the quantal conserved quantities in phase space (Li, 1996a,b).

## 4. NON-ABELIAN CS THEORY

To illustrate the use of the above results, we give a preliminary application to the non-Abelian CS theory.

The (2 + 1)-dimensional non-Abelian CS fields $A_\mu^a(x)$ coupled to the scalar field $\phi(x)$ was proposed by Kim and Park (1994) with a Lagrangian given by

$$\mathcal{L} = (D_\mu\phi)^+(D^\mu\phi) + \kappa\epsilon^{\mu\nu\rho}(\partial_\mu A_\nu^a A_\rho^a + \tfrac{1}{3}f_{bc}^a\, A_\mu^a A_\nu^b A_\rho^c) \tag{23}$$

where $\phi(x)$ is an $N$-component scalar field, and $D_\mu$ is the covariant derivative. The gauge invariance of the non-Abelian CS term requires the quantization of the dimensionless constant $k$ (Deser *et al.*, 1982).

We choose the Lorentz gauge

$$\partial^\mu A_\mu^a = 0 \tag{24}$$

and using the Faddeev–Popov trick to formulate the generating functional for this model, we obtain

$$Z[J, \eta^+, \eta, \bar{\xi}, \xi] = \int \mathcal{D}A_\mu^a\, \mathcal{D}\phi^+\, \mathcal{D}\phi\, \mathcal{D}\overline{C}_a\, \mathcal{D}C_a \exp\left[i\int d^4x\, (\mathcal{L}_{\text{eff}} + J_a^\mu A_\mu^a + \eta^+\phi + \phi^+\eta + \bar{\xi}C + \overline{C}\xi)\right] \tag{25}$$

where

$$\mathcal{L}_{\text{eff}} = \mathcal{L} + \mathcal{L}_{\text{fix}} + \mathcal{L}_{\text{gh}} \tag{26}$$

$$\mathcal{L}_{\text{fix}} = -\frac{1}{2\alpha_0}(\partial^\mu A_\mu^a)^2 \tag{27}$$

$$\mathcal{L}_{\text{gh}} = -\partial^\mu \overline{C}_a D_{b\mu}^a C_b \tag{28}$$

$\overline{C}_a(x)$ and $C_b(x)$ are ghost fields, and $D_{b\mu}^a = \delta_b^a\partial_\mu + f_{bc}^a A_\mu^c$. The effective Lagrangian is invariant under the following BRS transformation:

$$\begin{cases} \delta\phi = i\tau C_a T^a\phi \\ \delta\phi^+ = -i\tau C_a T^a\phi^+ \\ \delta A_\mu^a = -\tau D_{b\mu}^a C_b \\ \delta C^a = \tfrac{1}{2}\tau f_{bc}^a C_b C_c \\ \delta\overline{C}^a = -(1/\alpha_0)\tau\partial^\mu A_\mu^a \end{cases} \tag{29}$$

where $T^a$ are generators of the gauge group; moreover, $\delta(D_{b\mu}^a C_b) = 0$ and $\delta(\delta C^a) = 0$ under the transformation (29). Introducing the external sources

$U_a^\mu$ and $V^a$ with respect to $\delta A_\mu^a$ and $\delta C^a$, one can write the generating functional as

$$Z[J, \eta^+, \eta, \bar{\xi}, \xi, U, V]$$
$$= \int \mathcal{D}A_\mu^a\, \mathcal{D}\phi^+\, \mathcal{D}\phi\, \mathcal{D}\overline{C}_a\, \mathcal{D}C_a \exp\left[i \int d^4x\, (\mathcal{L}_{\text{eff}}\right.$$
$$\left. + J_a^\mu A_\mu^a + \eta^+\phi + \phi^+\eta + \bar{\xi}C + \overline{C}\xi + U_a^\mu \delta A_\mu^a + V^a \delta C_a)\right] \quad (30)$$

The generating functional (30) is invariant under the transformation (29), and the Jacobian of the transformation is equal to unity; thus, we have

$$\int \mathcal{D}A_\mu^a\, \mathcal{D}\phi^+\, \mathcal{D}\phi\, \mathcal{D}\overline{C}_a\, \mathcal{D}C_a$$
$$\times \left[\int d^4x\, (J_a^\mu \delta A_\mu^a + \eta^+\delta\phi + \delta\phi^+\eta + \bar{\xi}\delta C + \delta\overline{C}\xi)\right]$$
$$\times \exp\left[i \int d^4x\, (\mathcal{L}_{\text{eff}} + J_a^\mu A_\mu^a + \phi^+\eta + \eta^+\phi + \bar{\xi}C\right.$$
$$\left. + \overline{C}\xi + U_a^\mu \delta A_\mu^a + V^a \delta C_a)\right] = 0 \quad (31)$$

The Ward identity for Green functions follows as

$$\int d^4x \left[ iT^a\eta^+ \frac{\delta}{\delta\eta^+}\frac{\delta}{\delta\bar{\xi}^a} - iT^a\eta \frac{\delta}{\delta\eta}\frac{\delta}{\delta\xi^a} + J_a^\mu \frac{\delta}{\delta U_a^\mu} - \bar{\xi}^a \frac{\delta}{\delta V^a}\right.$$
$$\left. - \frac{1}{\alpha_0}\xi^a \partial_\mu\left(\frac{\delta}{\delta J_\mu^a}\right)\right] Z[J, \eta^+, \eta, \bar{\xi}, \xi, U, V] = 0 \quad (32)$$

The effective Lagrangian (26) is invariant under the transformation (29) and the Jacobian of the transformation (29) is equal to unity; according to expression (22) one obtains the conserved quantity at the quantum level

$$Q = \int d^2x \left[ -\frac{\kappa}{2}\epsilon_{ij}A^{ja}D_{bi}^a C_b + i(D_0\phi)^+ C_a T^a \phi - i\phi^+ C_a T^a (D_0\phi)\right.$$
$$\left. + (-\dot{\overline{C}}_a)\frac{1}{2} f_{bc}^a C_b C_c + \left(-\frac{1}{\alpha}\right) D_b^{0a} C^b \partial^\nu A_\nu^a \right] \quad (33)$$

The BRS transformation is nonlinear for ghost fields. Now we present the following nonlocal transformation (Li, 1995):

$$\begin{cases} \phi'(x) = \phi(x) + i\epsilon_0^\nu T^\sigma A_\nu^\sigma(x)\phi(x) \\ \phi^{+\prime}(x) = \phi^+(x) - i\epsilon_0^\nu T^\sigma A_\nu^\sigma(x)\phi^+(x) \\ A_\mu^{a\prime}(x) = A_\mu^a(x) + \epsilon_0^\nu D_{\sigma\mu}^a A_\nu^\sigma(x) \\ C^{a\prime}(x) = C^a(x) + i\epsilon_0^\nu (T_\sigma)_{ab} C^b(x) A_\nu^\sigma(x) \\ \overline{C}^{a\prime}(x) = \overline{C}^a(x) - i\epsilon_0^\nu \overline{C}^b(x)(T_\sigma)_{ba} A_\nu^\sigma(x) + \dfrac{i\epsilon_0^\nu}{\Box}\partial_\mu[\overline{C}^b(x)(T_\sigma)_{ba}\partial^\mu A_\nu^\sigma(x)] \end{cases} \tag{34}$$

where $\epsilon_0^\nu$ are parameters. It is easy to check that $\mathcal{L}$ and $\mathcal{L}_{gh}$ are invariant under the transformation (34) (Li, 1995). The Jacobian of the transformation (34) is equal to unity (Kuang and Yi, 1980). From the Ward identities (10) for the global transformation (34), we have

$$\begin{aligned} \int d^3x \bigg\{ & \frac{i}{\alpha}\partial^\mu\left(\frac{\delta}{i\delta J_\sigma^\mu}\right) - \partial^\rho D_{\sigma\rho}^a\left(\frac{\delta}{i\delta J_\sigma^\nu}\right) - \partial_\mu J_\sigma^\mu \frac{\delta}{\delta J_\sigma^\nu} - if_{\sigma c}^a J_\mu^a \frac{\delta}{\delta J_c^\mu}\frac{\delta}{\delta J_\sigma^\nu} \\ & + i\eta^+ T_\sigma \frac{\delta}{\delta J_\sigma^\nu}\frac{\delta}{\delta\eta^+} - i\eta T^\sigma \frac{\delta}{\delta J_\sigma^\nu}\frac{\delta}{\delta\eta} + \overline{\xi}_a (T_\sigma)_{ab}\frac{\delta}{\delta J_\sigma^\nu}\frac{\delta}{\delta\overline{\xi}_b} - \xi_a (T_\sigma)_{ba}\frac{\delta}{\delta J_\sigma^\nu}\frac{\delta}{\delta\xi_b} \\ & + \frac{1}{\Box}\partial_\mu\left[\frac{\delta}{\delta\xi_b}(T_\sigma)_{ba}\xi_a\partial^\mu\frac{\delta}{\delta J_\sigma^\nu}\right]\bigg\} Z[J, \eta^+, \eta, \overline{\xi}, \xi] = 0 \end{aligned} \tag{35}$$

As usual, let $Z[J, \eta^+, \eta, \overline{\xi}, \xi] = \exp\{iW[J, \eta^+, \eta, \overline{\xi}, \xi]\}$ and use the definition of generating functional of proper vertices $\Gamma[A, \phi, \phi^+, C, \overline{C}]$ which is given by performing a functional Legendre transformation of $W[J, \eta^+, \eta, \overline{\xi}, \xi]$; then the Ward identities (35) can written as

$$\begin{aligned} \int d^3x \bigg\{ & i\partial^\rho A_\rho^a \partial^\mu D_{\sigma\mu}^a A_\nu^\sigma - A_\nu^\sigma \partial_\mu \frac{\delta\Gamma}{\delta A_\mu^\sigma} \\ & - if_{\sigma c}^a A_\nu^\sigma A_\mu^c \frac{\delta\Gamma}{\delta A_\mu^a} + iA_\nu^\sigma \phi^a (T_\sigma)_{ab}\frac{\delta\Gamma}{\delta\phi^b} \\ & - iA_\nu^\sigma \phi^{+a}(T_\sigma)_{ab}\frac{\delta\Gamma}{\delta\phi^{+b}} + A_\nu^\sigma C^a (T_\sigma)_{ab}\frac{\delta\Gamma}{\delta C^b} - A_\nu^\sigma \overline{C}^a (T_\sigma)_{ba}\frac{\delta\Gamma}{\delta\overline{C}^b} \\ & + A_\nu^\sigma \partial^\mu\left[\partial_\mu\left(\frac{\delta\Gamma}{\delta\overline{C}^a}\frac{1}{\Box}\right)(T_\sigma)_{ba}\overline{C}^b\right]\bigg\} = 0 \end{aligned} \tag{36}$$

Functionally differentiating (36) with respect to $A_\mu^a(x_1)$, $\overline{C}^e(x_2)$, and $C^f(x_3)$ and setting all fields equal to zero, we obtain

$$\partial_{x_1}^\mu \frac{\delta^3\Gamma[0]}{\delta\overline{C}^e(x_2)\delta C^f(x_3)\delta A_\sigma^\mu(x_1)} - (T_\sigma)_{fb} \frac{\delta^2\Gamma[0]}{\delta C^e(x_2)\delta\overline{C}^b(x_1)} \delta(x_1 - x_3)$$
$$+ (T_\sigma)_{be} \frac{\delta^2\Gamma[0]}{\delta C^b(x_1)\delta\overline{C}^f(x_3)} \delta(x_1 - x_2)$$
$$+ \partial^\mu\left[\partial_\mu\left(\frac{\delta^2\Gamma[0]}{\delta\overline{C}^a(x_1)\delta C^f(x_2)} \frac{1}{\Box}\right)(T_\sigma)_{ea}\delta(x_1 - x_2)\right] = 0 \tag{37}$$

Functionally differentiating (36) many times with respect to the field variables, one can obtain various Ward identities for proper vertices.

The Ward identities (37) for gauge–ghost proper vertices differs from the Ward identities deriving from the BRS invariance for an effective Lagrangian in configuration space. The BRS transformation is nonlinear in ghost fields, while the transformation (34) is a linear (non-local) one. The above formulation to derive the Ward identities the invariant of the terms $\mathcal{L}$ and $\mathcal{L}_{\text{gh}}$ in (26) under the transformation (34) is only required. This is also different from BRS invariance for an effective Lagrangian.

The effective Lagrangian (26) is invariant under spatial rotation, and the Jacobian of the transformation for field variables is equal to unity; from (22) we obtain the quantal conserved angular momentum

$$J_{lk} = \int d^2x \left\{\left(-\frac{\kappa}{2}\epsilon_{ij}A^{ja}\right)\left(x_k\frac{\partial A_i^a}{\partial x_l} - x_l\frac{\partial A_i^a}{\partial x_k}\right) + \left(-\frac{\kappa}{2}\epsilon_{ij}A^{ja}\right)(\Sigma_{lk})_{i\nu}A_a^\nu\right.$$
$$+ D_0\phi^+\left(x_k\frac{\partial\phi}{\partial x_l} - x_l\frac{\partial\phi}{\partial x_k}\right) + \left(x_k\frac{\partial\phi^+}{\partial x_l} - x_l\frac{\partial\phi^+}{\partial x_k}\right)D_0\phi$$
$$+ (-\dot{\overline{C}}_a)\left(x_k\frac{\partial C_a}{\partial x_l} - x_l\frac{\partial C_a}{\partial x_k}\right) + \left(x_k\frac{\partial\overline{C}_a}{\partial x_l} - x_l\frac{\partial\overline{C}_a}{\partial x_k}\right)D_b^{a0}C^b \tag{38}$$

From this result we see that the conserved angular momentum at the quantum level differs from the classical one in that one needs to take into account the contribution of the angular momentum of ghost fields in non-Abelian CS theories.

## 5. CONCLUSIONS AND DISCUSSION

In this paper we have studied the quantal global symmetry for a gauge-invariant system in configuration space. The path integrals provide a useful tool. The phase-space path integrals are more fundamental than configuration-space path integrals. Based on phase-space path integrals the quantal canonical global symmetry for a system has been developed in previous work (Li, 1996a,b). For a gauge-invariant system a simpler and more useful method for the quantization is the Faddeev–Popov trick, from which the configuration-space generating functional can be derived; in certain cases this generating functional can also be obtained by carrying out explicit integration over the canonical momenta in the phase-space generating functional.

Starting from the configuration-space generating functional of the Green functions, the Ward identities for a gauge-invariant system under the global transformation are derived; the quantal conservation laws under the global symmetry transformation are also deduced if the effective action is symmetric and the Jacobian of the corresponding transformation in configuration space is equal to unity. In the general case these conservation laws differ from the classical ones.

The application of the theory to non-Abelian CS gauge fields coupled to scalar fields has been presented. The Ward identity for the BRS transformation and BRS-conserved quantity at the quantum level are derived. The quantal conserved angular momentum is obtained, which differs from the result derived by using the classical Noether theorem, because one needs to take into account the contribution of the ghost fields. Recent work (Antilon *et al.*, 1995) has studied the occurrence of fractional spin for non-Abelian CS theories in the classical case. We do not think these properties are always preserved for non-Abelian CS theories at the quantum level.

For Abelian CS theories, in the Lorentz gauge (or Coulomb gauge), the ghost fields are absent in the path-integral quantization using the Faddeev–Popov trick. One can proceed in the same way to conclude that the conserved angular momentum at the quantum level coincides with the result derived from the classical Noether theorem (Banerjee, 1994; Kim and Park, 1994). Thus, the fractional spin and statistics in Abelian CS theories (Banerjee, 1994; Kim and Park, 1994) are preserved in quantum theories.

We have presented here the quantal global symmetry in configuration space. All results for CS theories can also be derived by using the path integral of the canonical formalism as in previous work (Li, 1996a,b). This implies that the Faddeev–Popov trick is valid for these CS theories.

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## REFERENCES

Antilon, A., Escalona, J., and German, G., *et al.* (1995). *Physics Letters B,* **359**, 327.
Banerjee, R. (1993). *Physical Review D,* **48**, 2905.
Banerjee, R. (1994). *Nuclear Physics B,* **419**, 611.
Danilov, G. S. (1991). *Physics Letters B,* **257**, 285.
Deser, S., Jackiw, R., and Templeton, S. (1982). *Annals of Physics,* **140**, 372.
Dirac, P. A. M. (1964). *Lecture in Quantum Mechanics,* Yeshiva University, New York.
Faddeev, L. D. (1970). *Theoretical Mathematics and Physics,* **1**, 1.
Faddeev, L. D., and Popov, V. N. (1967). *Physics Letters B,* **25**, 29.
Faddeev, L. D., and Slavnov, A. A. (1980). Gauge Fields, The Benjamin/Cummings Publishing Company, Inc. London.
Joglekar, S. D. (1991). *Physical Review D,* **44**, 3879.
Kim, W. T., and Park, Y.-J. (1994). *Physics Letters B,* **336**, 376.
Kuang, Y.-P., and Yi, Y.-P. (1980). *Physica Energiae fortis et Physica Nuclearis,* **4**, 286.
Lerda, A. (1992). Anyons, Springer-Verlag, Berlin.
Li, Z.-P. (1993). *Classical and quantal dynamics of the constrained system and their symmetry properties,* Beijing Polytechnic University Press, Beijing.
Li, Z.-P. (1994). *Physical Review E,* **50**, 876.
Li, Z.-P. (1995). *International Journal of Theoretical Physics,* **34**, 523.
Li, Z.-P. (1996a). *International Journal of Theoretical Physics,* **35**, 1353.
Li, Z.-P. (1996b). *Science in China A,* **26**, No. 6.
Mizrahi, M. M. (1978). *Journal of Mathematical Physics,* **19**, 298.
Nishikawa, T.-I. (1993). *Physics Letters,* **B**, 309, 351.
Noether, E. (1918). *Nachrichten Gesellschaft der Wissenschaften Gottingen Mathematisc Physicalishe Klasse,* **1918**, 235.
Senjanovic, P. (1976). *Annals of Physics,* **100**, 227.
Young, B.-L. (1987). *Introduction to Quantum Field Theories,* Science Press, Beijing.